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Gauge Points and their Advantages

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In a previous article[1] I have already mentioned gauge points (gp) as special calculation aids when working with the slide rule. The historical literature mentions numerous other gauge points for calculations in the fields of geometry, stereometry, mechanics, conversion of measures and weights, interest calculations as well as in engineering for the design of machines and machine parts. In this paper, selected examples are used to give more details on these helpful numbers with their special features and advantages.

What are Gauge Points?

They give constant numerical values, which are entered in a calculation with the slide rule as points on the scale. The Dictionary of Arts and Sciences[2], issue 1763, explains:

> GAUGING, the art or act of measuring the capacities or contents of all kinds of vessels, and determining the quantities of fluids or other matters contained therein.

In stereometry, the gauge points have their origin. In this dictionary the term gauge point is very narrowly defined and refers only to the conversion of measure units for volumes:

> *GAUGE-POINTS, of a solid measure, the diameter of a circle, whose area is equal to the solid content of the same measure.*

The background of this difficult-to-understand definition is explained in more detail below in example 1.

In the following period, pre-calculated numerical values for other applications have also been referred to as gauge points. German-language literature mostly uses the term “divisor” for this purpose.

The authors of the examined textbooks used explanations in different extents for gauge points. Some just show their use, but do not explain them. Others use general terms for description. Flower 1768[3] only distinguishes:

> ...it is moreover necessary, that the reader should know when the principal agent in any proposition be a factor, divisor or gauge-point; also, by what particular lines of the instrument the said proposition is to be solved,...

In 1875 Dixon[4] writes more accurately in the explanation of the use of the slide rule:

> ...attention is next directed to what are usually termed ‚constants‛, or ‚gauge points‛, multipliers, and divisors, for reducing figures of one denomination to another...

He continues:

> … and it is essential, in using Slide Rules for calculations, to have numerous well arranged authentic tables of ‚Constants‛, or ‚Gauge Points‛, referring to Money, Weight, Measure, Time, &c., &c,. in order to obtain speedily, practical answers.

Later in his book, Dixon shows that gauge points can operate on more than just conversions. Other authors also provide detailed explanations for the calculation and application with gauge points. I refer to them in the examples.

The use of gauge points is closely related to calculations in proportions. One must keep in mind that the slide rule is a device for solving proportions. The Dictionary of Arts and Sciences writes about this on page 1405:

> In multiplication, either of whole numbers, mixed or decimal fractions, the proportion is, as 1: the multiplicator ::(equals)the Multiplicand : the product. Example. Let it be required to multiply 6 by 4. The proportion then is as 1:4::6:24. Therefore set 1 upon the line B, to 4 upon the line A; then against 6 upon B, is 24, the product sought, upon A.

A corresponding procedure is shown for the division. The principle of proportions can also be clearly seen in the simple example for calculating circumference or diameters of a circle with the approximation pi ≈ 22/7 in Figure 1[5]. The proportion here is 22/7 = circumference / diameter.

9. Circumference and Diameter of Circle (on A and B).

A	Set 22	Line of circumferences
B	To 7	Line of diameters.

FIGURE 1. Calculation of the Circumference or Diameter of a Circle

This representation as a proportion offers an advantage, because in a single position of the slide rule not only two values, in this case circumference and diameter, but also their dependencies can be read off.

A distinction has yet to be made. Gauge points and gauge marks are very similar in use. Venetsianos[6] has collected gauge marks in a comprehensive list. To simplify, one could distinguish that gauge marks are marked on the scales, while gauge points have been collected and published in external tables. This distinction cannot be drawn sharply; just think of the points wine gallon (WG), ale gallon (AG), and others, which appear in tables, and are sometimes marked with small brass pins on the scales too.

The following examples describe selected gauge points and calculate their values. A glance at the slide rule setting clarifies the reading of the result. We also will realize that the historical English system of units was neither homogeneous nor coherent and from today's point of view needs training to be understood.

Example 1: The Gauge Point Wine Gallon

Two informative explanations can be found at Symons[7] on page 54f and in the previously mentioned Dictionary of Arts and Sciences on page 1407. The starting point is the question of how to determine the volume of a vessel with a circular cross-section in wine gallons (WG). A WG equals 231 in^3.

A circle with 1 inch diameter has an area of pi / 4 = 0.785 in^2. Now the authors argue that this is as much as 0.785 / 231 WG. This change in the unit from in^2 to in^3 can only be understood by multiplying the result by 1 inch. This action creates a cylinder with the diameter 1 and height 1 and the volume 0.785 / 231 WG. If the cylinder has a diameter ≠ 1 and a length l ≠ 1, then the volume is $V = d^2 * l * 0.785 / 231$ WG.

The calculation of the volume of a cylindrical vessel with diameter d = 27 inches and length L = 40 inches is made on a slide rule for gauging as shown in Figure 2. The upper scale extends from 1 to 3.16 ($=\sqrt{10}$), the lower one on the slider from 1 to 10. Opposite the marked point WG on the upper scale, the length 40 is set on the lower scale. Against the point of diameter 27, in the upper scale, the volume 99.1 wine gallons can be read below.

The proportional calculation here is $WG^2 / L = d^2 / V$. Compared with the formula for the volume, the result is $WG = \sqrt{4 \times 231 \div \pi} = 17.154$. This is the value of the gauge points wine gallon for cylindrical vessels. One can also say that a cylinder with a diameter WG (= 17.154) inches and a height of 1 inch has a volume of WG (= 231) in^3. This is the explanation of the sentence cited above in the Dictionary of Art and Sciences.

The gauge point replaces three steps in calculation: two multiplications and one division. The volume looked for can be read directly.

For vessels with square cross-section or for other volume units, other values for the gauge point are necessary. Figure 3, taken from Symons, shows an adjusted table. The table contains a print error in the fourth line from top and should read*, 7854) 2150.4200 (2737.99).* The value for the gauge point is given correctly. The gauge points wine gallon and ale gallon are the oldest ones. William Oughtred used them in 1633 in his book *The new artificial gauging line or rod* for the calculation and conversion of volumes. Over time, the term gauge points has been retained while their use has expanded significantly.

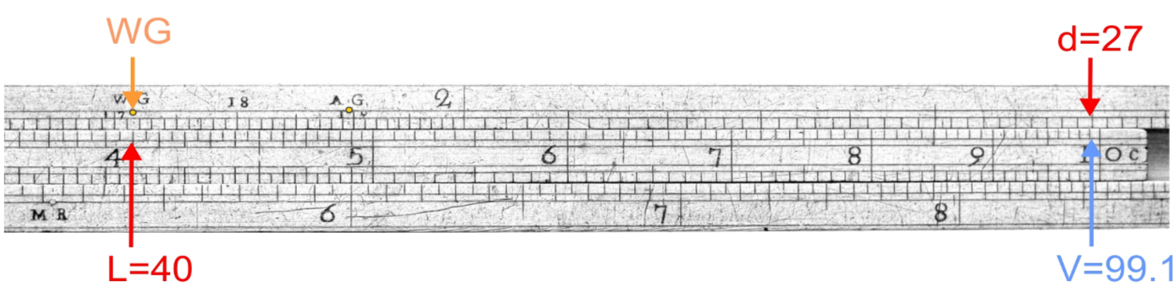

FIGURE 2. Setting the Slide Rule for Wine Gallons

Of finding Factors, &c. 55

See the work.

Div^r.	Div^d.	Quot^ts.	S. R^ts.	
,7854)	282.0000 (	359.05	18.95	Ale gallons.
,7854)	231.0000 (	294.12	17.15	Wine gallons.
,7854)	268.8000 (	342.24	18.5	Malt gallons.
,7854)	2150.4200 (	737.92	52.32	Malt buſhels.
,7854)	227.0000 (	289.	17	Maſh tun gall.

FIGURE 3. Table of the Gauge Points for Cylindrical Vessels

Example 2: Polygons with their Circumscribed Circle

As an example in geometry, we look at polygons in their circumscribed circle. Dixon gives the table (See Figure 4) on page 82.

For the calculation of the diameter of a circle circumscribed to the polygon, the table (See Figure 4) gives in the third line from the top the gauge points as a function of the number of sides of the polygon, noted in the line above.

If we denote with n the number of sides of the polygon, with d the diameter of the circumscribed circle and with s the length of one side of the polygon (See Figure 5), then we get $d = s / \sin (180 / n)$, or written in a proportion $d / s = gp / 1$ with $gp = 1 / \sin (180 / n)$ for the gauge point (gp).

The corresponding setting of the slide rule is shown in Figure 6. Here and in the following examples, a slide rule of type Soho is used with the scales A = B = C = 1−10−100 and D = 1−10. As a template for the slide rule image the picture from Farey[8], page 537 is used.

By setting the start of scale B to the gauge point on A one can immediately read side lengths and associated diameters on A and B. Here the gauge point replaces two divisions and the determination of a value of the trigonometric function sin.

Example 3: Weight of a Globe

Howe[9] gives the following task on page 23 in his collection of exercises with the slide rule:

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IV.- How many pounds will a solid globe of brass weigh, 6 inches in diameter? The gauge is 637. Set 6 upon B to 637 upon A and against 6 upon D are 34 lbs, the answer, upon C
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(See Figure 7).

Earlier in the text, the author explains how to set such a type of task as a proportion and how to use the necessary gauge point:

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The general rule for a globe is: as the gauge point on A is to the diameter on B, so is the content on C to the diameter on D; or bring the diameter upon B to the gauge point upon A, and against the diameter upon D you have the content, or answer, upon C.
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The setting of the slide rule is shown in Figure 7.

With the diameter d of the globe in inches, its weight w in pounds, and the specific weight b in lbs/in^3 of the material, the recalculation gives the following relationships: $w = pi / 6 * b * d^3$ or $w * gp = d^3$. In a proportion for the slide rule it is written $w / d^2 = d / gp$ with $gp = 1 / (b * pi / 6)$. With a specific weight of brass at b = 0.3 lbs/in^3, this results in gp = 6.37 [in^3/lbs]. The values of gauge points are often given not as absolute values but as numbers.

Two features are immediately apparent: the gauge point combines both a mathematical constant and the specific weight of the material, and the third power of the diameter is converted into the product $d^2 * d$. With this technique, a scale of third power is dispensable.

The literature gives numerical values for gauge points depending on the material, i.e., for the specific weight and the shape of the body. These include globes, cylindrical and cuboidal bodies with one, two, or three dimensions. The dimensions may be measured in units of feet (F) or inches (I) or a combination of both, denoted FFF, FII, III. The units of measure used are essential because the values of the gauge points depend on them. Howe gives such a table on page 35 (See Figure 8).

Similar tables are sometimes found on two-legs folding slide rules.

APPLICATION OF TABLE OF POLYGONS. 83

Names.	Trigon	Square	Penta-gon.	Hexa-gon.	Hep-tagon.	Octa-gon.	Nona-gon.	Deca-gon.	Harde-cagon.	Dode-cagon.
No. of Sides.	3	4	5	6	7	8	9	10	11	12
iam. of least ircumscribing ircles	1·155	1·414	1·701	2·	2·305	2·613	2·924	3·236	3·55	3·864
Areas.	·433	1·	1·72	2·598	3·634	4·828	6·182	7·694	9·366	11·196
iam. of Circles f equal areas o Polygons ...	·7425	1·1283	1·48	1·819	2·151	2·479	2·805	3·13	3·453	3·775

FIGURE 4. Gauge Points for Polygons

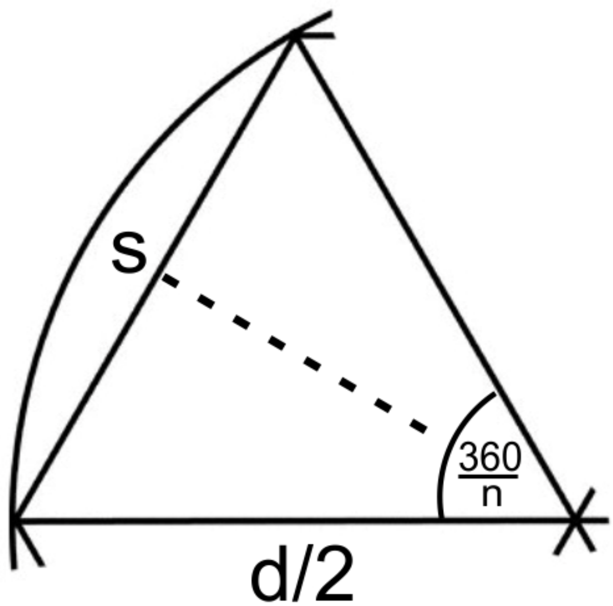

FIGURE 5. Section of a Regular Hexagon (n = 6)

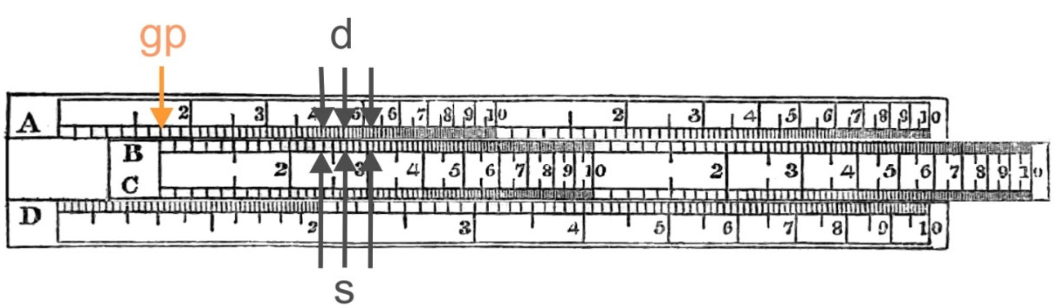

FIGURE 6. Setting of the Slide Rule for Example 2 with n = 6

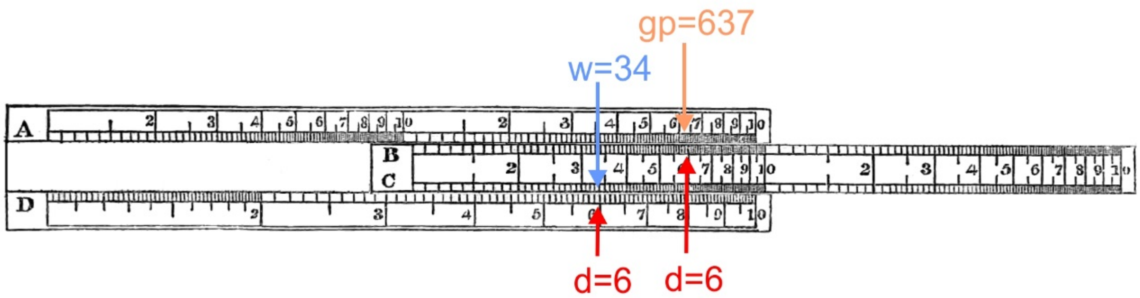

FIGURE 7: Setting the Slide Rule for Determining the Weight of a Globe

Example 4: Design of a Steam Engine

The Industrial Revolution in England from the second half of the 18th century is inextricably linked to the invention of the steam engine. Technical textbooks soon gave instructions for their design.

Farey writes in his textbook from 1827 on page VII:

> One great object of the present work is to furnish practical engineers with a series of rules for calculating all proportions, and quantities, which can be required to be known for the construction and use of steam-engines;...

and on the next page:

> The method of performing each calculation by the slide rule is added, and will tend to facilitate the computations.[10]

One of the rules for the design of a steam engine by Farey is singled out here. On page 575 he writes:

> Multiply the number of horse power by the constant number 6050; divide the product by the motion of the piston, in feet per minute, and the square root of the quotient is the proper diameter for the cylinder of the engine in inches.

The basics of this rule of thumb and its variants were derived from the data of machines already built, which are listed in a table on page 574. For calculations with this rule, Farey gives the setting of a slide rule shown in Figure 9. Here only the scales C and D are used.

This results in the proportion *(motion piston)* / gp^2 = *HP* / $diam^2$ and thus *HP* = *(motion piston)* * $diam^2$ / gp^2.

Farey also explains the derivation of the value for the gauge point 77.78 = $\sqrt{6050}$. In short, he starts with the experience that for 1 horse power (HP) a steam flow of about 33 ft^3 per minute is needed. This volume is equivalent to the volume 6050 cylindrical inch feet[11]. He points out that this value must be slightly adapted to the expected power of the machine.

Gauge points can be of dimension 1 (gp [1]) or dimensioned. The authors of textbooks do not address this feature. The users of the gauge points did not even think about it. They knew which numbers to put in the proportions and which unit is bound to the result. The gauge point used in this example has the unit gp^2 [in^2/lb], here with lb as pound force. Substituting this dimension in the above mentioned proportion shows that it is adequate for the physical relationship *power = force * speed*.

For the machine designer, such specifications result in a reduction in their engineering activity because with help of the gauge points a relatively complex dependence of three variables is reduced to a proportional calculation on the slide rule. The variables here are the power of the machine, the diameter of the cylinder, and the distance traveled by the piston in one minute. The engineer became able to easily vary the data of the machine with help of the slide rule and without intermediate calculations study their dependencies. Only then did he determine the data according to other specifications and began calculating the strength of the components.

I have discussed logarithmic calculators especially made for the design of piston engines and for the efficiency of a steam boiler in two earlier articles[12].

For steam engine component parts, as well as for boilers and their firing systems, there also existed construction rules as proportions based on gauge points[13]. The same applies to water pumps[14], because in search of coal for the steam engines mines were dug deeper and deeper and increasingly groundwater was encountered.

Wrong Gauge Points

One problem that affects all numerical values listed in tables are the errors they contain. In contrast to logarithmic tables, which were examined and compared in detail, there are only a few contributions to errors in tables with gauge points.

In an essay for IM2017 van Poelje[15] describes three circular slide rules with their identical tables of gauge points containing errors of 10% to 20%. In addition, these tables, including their errors, were also copied into an instruction manual for a slide rule by Routledge. Another source quoted by the author states that the tables at Routledge are said to be interspersed with errors and have survived in this state for nearly a century.

A TABLE OF GAUGE POINTS.

For weighing the different articles contained therein.

	SQUARE.			CYLINDER		GLOBE.	
	FFF	FII	III	FF	II	F	I
Oak.	174	252	303	320	386	332	578
Mahogany.	15	217	2605	276	333	286	49
Box.	155	243	269	31	342	296	512
Red Deal	242	351	422	458	539	461	805
Marble	591	85	102	116	13	113	195
Brick.	795	115	138	147	176	152	263
Oil	174	25	301	319	383	332	574
Bees' Wax.	16	231	278	294	355	306	53
Sulphur.	8	115	138	146	126	153	264
Alcohol.	193	278	333	354	424	369	637
Air.	128	1843	22118	2347	28162	244	42243
Malt Bushels.	125	179	2150	2276	2738	267	41

FIGURE 8. Gauge points for Solid Bodies at Howe

Sliding Rule.	C	Motion of piston ft. per min.	Horse power of eng.
	D	Gage point 246 or 778.	Diam. of cylind. inc.

FIGURE 9. Example for the Design of a Steam Engine at Farey

Notes

1. Weiss, Stephan, *The Methodology of Teaching a Logarithmic Slide Rule in Historical Sequence,* Journal of the Oughtred Society , 26:2, Fall 2017.
2. A New and Complete Dictionary of Arts and Sciences, 2nd ed., v.2, London, 1763.
3. Flower, William, *A Key to the modern Sliding Rule,* London, 1768, p. X with tables in chap. VII.
4. Dixon, Thomas, *Treatise on the Arrangement, Application, and Use of Slide Rules*, *for Purposes of Engineers', Mechanics' and other Calculations,* Bradford, 1875, page 53.
5. Hoare, Charles, *The Slide Rule and How to Use It*, London, 1868, page 18.
6. Venetsianos, Panagiotis, *Pocketbook of the Gauge Marks,* 2nd edition, The Oughtred Society, 2011.
7. Symons, William, The Practical Gager, London, 1777.
8. Farey, John, *A Treatise on the Steam Engine,* London, 1827.
9. Howe, Joseph, I*nstructions and a Practical Treatise on the Improved Slide Rule,* London, 1845.
10. Starting in 1775, Watt and Boulton introduced improved slide rules, the so-called Soho type, in their factory in Soho, near Birmingham, for the designs of their machines (see Farey, pp. 531 and 536, as well as Wess, Jane, *The Soho Rule,* Journal of the Oughtred Society, 6:2, Fall 1997 and Wyman, Thomas, *Soho Steam Engines The First Engineering Slide Rule and the Evolution of Excise Rules*, Journal of the Oughtred Society, 22:2, Fall 2013). From then on, the slide rule became an indispensable tool for engineers.
11. For the units used here in detail: a circle with the diameter d inches has an area of $d^2 * pi / 4$ in^2 or an area of d^2 circular inches. It applies *1 circular inch = pi / 4 in*^{2}. Multiplying this basic area by the height h in inches results in a volume of $h * d^2 * pi / 4$ in^3 or $d^2 * h$ cylindrical inches. If the height is measured in feet, the multiplication gives cylindrical inch foot. The effective pressure of the steam is assumed to be 6.944 pounds per square inch and 5.454 pounds per circular inch, respectively.
12. Weiss, Stephan, *Golding's Horse Power Computer (1908)*, Journal of the Oughtred Society , 23:2, Fall 2014. Weiss, Stephan, *Crompton - Gallagher: Boiler Efficiency Calculator (1919)*, Journal of the Oughtred Society, 25:2, Fall 2016.
13. Armstrong, Robert, *An Essay on the Boilers of Steam Engines: their Calculation, Construction, and Management...*, London, 1839.
14. See the recalculation of a steam driven water pump in my previous article in Figure 2 and footnote 8 and other examples at Coulson, S,: *Coulson's Treatise on his newly invented Engineers' and Mechanics' Slide Rule,* Stokesley, 1842, page 229: On Pumps and Pumping Engines.
15. Van Poelje, Otto E., *Three of a Kind*. In: Kleine, Karl (editor), *Calculating in Everyday Life*, Proceedings 23rd International Meeting of Collectors and Researchers of Historical Computing Instruments (IM2017), Bonn, Germany.