Successors of Slonimsky's Multiplying Device 1844

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In 2011 I wrote an article about Hayyim Selig Slonimsky[1] (1810–1904), his theorem and his multiplying device, the latter he introduced in 1844 [11]. For every multiplication table that gives products of an arbitrary multi-digit number multiplied by 2, by 3, up to 9 only 280 unique columns exist and two indices within the range 1 to 28 for connection between them are necessary. This system avoids difficulties in performing a tens carry during calculations. The whole set of columns and their connections, arranged and published by the mathematician August Leopold Crelle, is shown in the appendix. Crelle also gave a proof of Slonimsky's theorem in 1848 [5]. For derivation of this system and its usage I refer to my previous article.

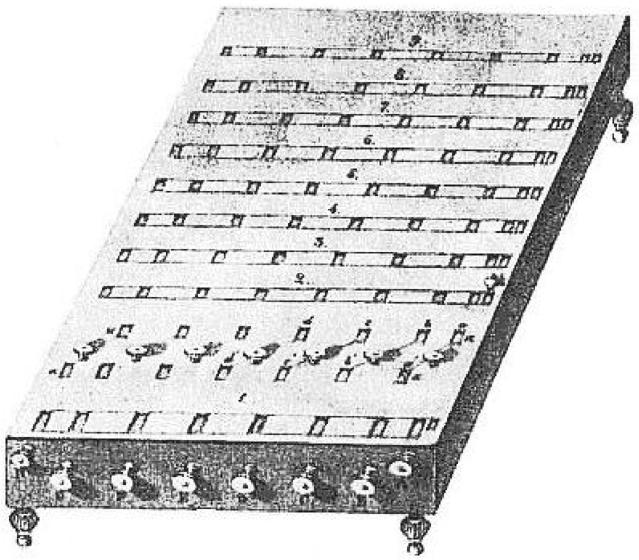


Fig. 1: Slonimsky's Multiplying Device

Slonimsky himself was the first who designed a multiplying device based on these 280 columns. They are written onto turnable cylinders, mounted side by side in a frame (see fig. 1). Later inventors took a different approach.[2]

1 Also written Slonimski or Slonimskii.

2 In the years 1846 and 1847 David Barnett from Birmingham were granted two patents for

Knight 1847

Only one year after publication of Crelle's complete table Henry Knight published his *Multiplication Tablets* [8]. Here he uses the same arrangement as Crelle with a separated page for every digit (fig. 2).
The booklet is extremely rare, perhaps due to the fact that it contains 3 errors.

3

	0	1	2	3	4	5	6	7	8	9	
1	0	3	6	9	2	5	8	1	4	7	9
2	0	3	6	9	2	5	8	1	4	8	9
3	0	3	6	9	2	5	8	1	5	8	9
4	0	3	6	9	2	5	8	2	5	8	9
5	0	3	6	9	2	5	9	2	5	8	9
6	0	3	6	9	2	6	9	2	5	8	9
7	0	3	6	9	2	6	9	2	5	9	9
8	0	3	6	9	3	6	9	2	6	9	9
9	0	3	6	9	3	6	9	3	6	9	9
10	0	3	6	0	3	6	0	3	6	0	10
11	0	3	6	0	3	6	0	3	7	0	10
12	0	3	6	0	3	7	0	3	7	0	10
13	0	3	6	0	3	7	0	4	7	0	10
14	0	3	6	0	3	7	0	4	7	1	10
15	0	3	7	0	4	7	1	4	8	1	10
16	0	3	7	0	4	7	1	4	8	2	10
17	0	3	7	0	4	7	1	5	8	2	10
18	0	3	7	0	4	8	1	5	8	2	10
19	0	3	7	0	4	8	1	5	9	2	10
20	0	3	7	1	4	8	2	5	9	3	10
21	0	3	7	1	4	8	2	6	9	3	10
22	0	3	7	1	5	8	2	6	0	3	11
23	0	3	7	1	5	8	2	6	0	4	11
24	0	3	7	1	5	9	2	6	0	4	11
25	0	3	7	1	5	9	3	6	0	4	11
26	0	3	7	1	5	9	3	7	0	4	11
27	0	3	7	1	5	9	3	7	1	4	11
28	0	3	7	1	5	9	3	7	1	5	11
	0	1	2	3	4	5	6	7	8	9	

3

Fig. 2: Table for digit 3 at Knight 1847

his calculating instrument [2, 3]. This instrument looks quite similar to that of Slonimsky and it works with parallel mounted turnable cylinders too, but the used system differs from Slonimsky's theorem.

Filipowski 1860

The next inventor in our sequence is Herschell Filipowski[3] (1816-1872). He worked as Hebraist as well as an actuary and seems to have been a successful writer, translator and publisher in both professions. With regard to mathematics he translated Napier's *Wonderful Canon* from Latin to English [9], published a logarithmic table [6], a Table of Excange [7] and various articles about actuarial mathematics. The Jewish Encyclopedia provides additional information about his life and work. The printed label inside the lid of a box containing his calculating rods, notes
'*N.B. — H. Filipowski gives lessons in Mathematics and Astronomy; also in the German and Oriental Languages, at very moderate terms. London, 25, Wilson streel, Finsbury-square*'.

About 1860 he designed and published a multiplying device, shown in fig. 3.

Fig. 3: Set of Filipowski's Calculating Rods

3 His name varies. It is written *Hirsch Filipowski (Phillip)* in the Jewish Encyclopedia (http://www.jewishencyclopedia.com), *Herschell E. Filipowski*, mostly used in his mathematical publications, or simply *H. Filipowski* (see *Mathematics of Computation*, V. 3, N. 21 (1948) p. 67). An article in *The Assurance Magazine, and Journal of the Institute of Actuaries,* Vol. 4, No. 3 (1854), p. 245, he signs with *Herschel Filipowski*.

It includes a set of 56 cylindrical rods, made of wood and stored in holes in a wooden case with overall dimensions 128 mm x 130 mm x 75 mm. Up to 8 rods can be arranged side by side in a slim wooden tray in order to build the desired multiplication table. Each rod shows ten columns with digits and letters printed on paper and bears a knob on its top. The knob is marked with a single letter.

The underlying principle of this device can easily be determined when we follow the calculating example Filipowski gives on the printed label inside the box. He completes his description with a table showing the aimed result, which is reproduced below.
Required are the first nine multiples of number 37582. For units the rod with its so called initial letter A on top of its knob has to be used in every calculation. The rod is set into the tray on the right side and turned until figure 2, the unit in 37582, shows up. Now on top the column initial B can be read. According to this letter a rod with a B on its knob is set next on the left side and turned until the tens figure 8 is visible. The new column initial H marks the next rod to be used and so on. Filipowski's explanatory table added to his example looks like this:

The rod initials	Q	X	U	H	B	A
The column initials	A	Q	X	U	H	B
The figures proposed	0	**3**	**7**	**5**	**8**	**2**
Second multiple	0	7	5	1	6	4
Third multiple	1	1	2	7	4	6
Fourth multiple	1	5	0	3	2	8
Fifth multiple	1	8	7	9	1	0
Sixth multiple	2	2	5	4	9	2
Seventh multiple	2	6	3	0	7	4
Eighth multiple	3	0	0	6	5	6
Ninth multiple	3	3	8	2	3	8

From this example it is evident, that Filipowski uses Slonimsky's theorem, although Slonimsky isn't mentioned at all in the explanatory text. The only modification concerns the replacement of indices by letters. In comparison with Crelle's arrangement each rod bears those ten rows of multiples, arranged as columns, that are given in a single horizontal line at Crelle. The bold index number at Crelle on the left and right side of a line is replaced here by a letter on top of each knob. Finally a column initial points to the next rod to be used.

Fifty-six rods bear 560 = 2 x 280 columns, which therefore hold two complete sets of columns. From the given example the assignments A≡1, B≡6, H≡24, Q≡11, U≡17, X≡22 can be derived. Further assignments remain unsolved at the moment, especially it is unknown which signs he uses to separate 28 different rods with only 26 letters. The provided images don't allow a closer look.

Furthermore he mentions two rods 'c' and 'd' which should serve as substitutes whenever a repetition of one and the same initial may occur.

Up to now two sets of Filipowski's rods are known to me: one at the Science Museum, London, from where fig. 3 is copied. The other set has been sold for £ 4400 at the auction Dominic Winter, Gloucestershire, UK, held at April 10th, 2013.

Joffe 1881

Only little is known about Joffe. In the Jewish Encyclopedia he is named Zebi Hirsch Jaffe[4]. He has been a Russian mathematician and writer. Among others the Encyclopedia tells us

> 'He received the usual Talmudic education and early showed extraordinary mathematical talent. His father would not allow him to enter a public school, and, not having the opportunity to study mathematics from books, Jaffe began to solve algebraic problems according to rules of his own discovery. In 1873 his father presented him with Ḥayyim Selig Slonimsky's works as well as with other mathematical works in Hebrew. In 1877 Jaffe published in "Ha-Ẓefirah" (No. 24) his first mathematical article, and since that time he has contributed many mathematical and Talmudic articles to that periodical and to "Ha-Asif." In 1881 Jaffe went to Moscow, where he exhibited his calculating-machine, which won him honorary mention by the administration of the exhibition.'

It should be pointed out that he obviously has been familiar with Slonimsky's work. His success at the All-Russian exhibition in 1882 is also mentioned by Apokin [1].
Later indications of the rods are not known to me.

Joffe places Slonimsky's resp. Crelle's columns onto 280 sides of 70 square rods. Bohl [4] presents four sides on four rods (fig. 4a). The figures to build a multi-digit number are set on top, in fig. 4a this number is 0325. Combinations of uppercase letters with Roman numerals are placed below and at the bottom of a side. In comparison with Crelle's arrangement these specific signs represent the indexes 1≡IA, 2≡IIA, ..., 7≡VIIA, 8≡IB, 9≡IIB, ..., 14≡VIIB, ..., up to 28≡VIID. We can assume that he used combined signs, because they are easier visually perceived than pure numbers and therefore prevent errors.
For the first rod of units on the right side always rod IA must be used. The sign at the lower end indicates the set of rods to be used left of it for the tens and so on.

In the image at Bohl the rods are positioned in an U-shaped frame with inscriptions x2, x3,... down to x9. The frame eases positioning of rods and reading of multiples but it isn't strictly necessary.

4 Also written Iofe or Jofe. I thank Georgi Dalakov from http://history-computer.com/ for pointing me to the entry in the Jewish Encyclopedia.

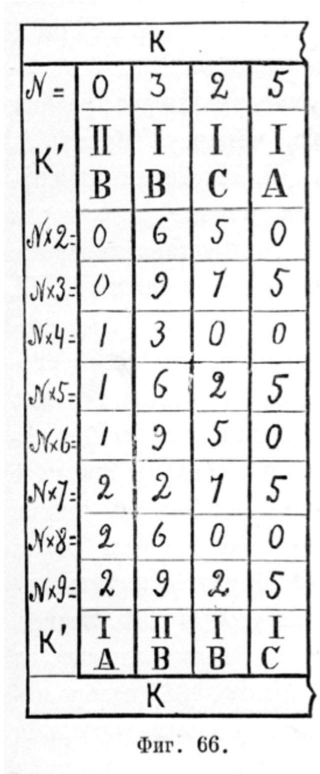
Like other devices based on Slonimsky the rods replace the labour of tens carry by only selecting rods. On the other hand to get an overview of 70 rods with 280 sides needs some organising. Moreover 70 rods will hardly be enough. Since each column is represented only once and with use of a single rod 3 sides are no longer available we must assume that much more than a single set with 70 rods is necessary to build the multi-digit number.

Now having reconstructed the design of Joffe's rods, the whole set can be re-created. For that purpose I scanned Crelle's table with tools for optical character recognition, checked the figures with software written for that purpose by my-self. With help of the graphical environment *Processing*[5] and a few hours pro-gramming a printout of 70 rods with their 4 sides has been created. Fig. 4b shows two reconstructed rods. In a last step the prints need to be cut and pasted around wooden rods from a do-it-yourself store.

In 1891, ten years after Joffe having presented his invention, Genaille and Lucas solved the problem of automatic tens carry much better with less different rods [10].

A final addition should be made to avoid confusion, found now and then in de-scriptions. The mentioned rods, designed by Filipowski and Joffe, are by no means derived from Napier's rods although they look quite similar. Whereas with Napier's rods the user has to perform a tens carry by himself, here the tens carry is already implemented within the columns.

5 URL http://processing.org/



K				
N =	0	3	2	5
K'	II B	I B	I C	I A
N×2 =	0	6	5	0
N×3 =	0	9	7	5
N×4 =	1	3	0	0
N×5 =	1	6	2	5
N×6 =	1	9	5	0
N×7 =	2	2	7	5
N×8 =	2	6	0	0
N×9 =	2	9	2	5
K'	I A	II B	I B	I C
K				

Фиг. 66.

Fig. 4a: A Joffe's rod in Bohl 1896

5	5	5	5
VII C	I D	II D	III D
1	1	1	1
7	7	7	7
2	3	3	3
8	8	8	9
4	4	4	4
0	0	0	0
5	6	6	6
1	1	2	2
III C	III C	III C	III C

5	5	5	5
IIII D	V D	VI D	VII D
1	1	1	1
7	7	7	7
3	3	3	3
9	9	9	9
5	5	5	5
0	1	1	1
6	6	7	7
2	2	2	3
III C	III C	III C	III C

Fig. 4b: Two Joffe's rods from reconstructed set

Appendix: Complete set of 280 different columns in multiplication tables

Source: Crelle, *Journal* 1846 [5]

No.	0: 01234 56789	No.	1: 01234 56789	No.	2: 01234 56789	No.	3: 01234 56789	No.	4: 01234 56789	No.
1	00000 00000	1	01234 56789	1	02468 02468	6	03692 58147	9	04826 04826	12
2	00000 00001	1	01234 56780	2	02468 02469	6	03692 58148	9	04826 04827	12
3	00000 00011	1	01234 56790	2	02468 02479	6	03692 58158	9	04826 04837	12
4	00000 00111	1	01234 56890	2	02468 02579	6	03692 58258	9	04826 04937	12
5	00000 01111	1	01234 57890	2	02468 03579	6	03692 59258	9	04826 05937	12
6	00000 11111	1	01234 67890	2	02468 13579	6	03692 69258	9	04826 15937	12
7	00000 11112	1	01234 67891	2	02468 13570	7	03692 69259	9	04826 15938	12
8	00001 11122	1	01235 67801	3	02469 13580	7	03693 69269	9	04827 15948	12
9	00001 11222	1	01235 67901	3	02469 13680	7	03693 69369	9	04827 15048	13
10	00011 12223	1	01245 68902	3	02479 14681	7	03603 60360	10	04837 16049	13
11	00011 12233	1	01245 68912	3	02479 14691	7	03603 60370	10	04837 16059	13
12	00011 22233	1	01245 78912	3	02479 24691	7	03603 70370	10	04837 26059	13
13	00011 22333	1	01245 78012	4	02479 24791	7	03603 70470	10	04837 26159	13
14	00011 22334	1	01245 78013	4	02479 24792	7	03603 70471	10	04837 26150	14
15	00112 23344	1	01346 79023	4	02570 25702	8	03704 71481	10	04938 27160	14
16	00112 23345	1	01346 79024	4	02570 25703	8	03704 71482	10	04938 27161	14
17	00112 23445	1	01346 79124	4	02570 25803	8	03704 71582	10	04938 27261	14
18	00112 33445	1	01346 89124	4	02570 35803	8	03704 81582	10	04938 37261	14
19	00112 33455	1	01346 89134	4	02570 35813	8	03704 81592	10	04938 37271	14
20	00122 34456	1	01356 80135	5	02580 36814	8	03714 82593	10	04948 38272	14
21	00122 34556	1	01356 80235	5	02580 36914	8	03714 82693	10	04948 38372	14
22	00123 34566	1	01357 80245	5	02581 36924	8	03715 82603	11	04949 38382	14
23	00123 34567	1	01357 80246	5	02581 36925	8	03715 82604	11	04949 38383	14
24	00123 44567	1	01357 90246	5	02581 46925	8	03715 92604	11	04949 48383	14
25	00123 45567	1	01357 91246	5	02581 47925	8	03715 93604	11	04949 49383	14
26	00123 45667	1	01357 91346	5	02581 47025	9	03715 93704	11	04949 49483	14
27	00123 45677	1	01357 91356	5	02581 47035	9	03715 93714	11	04949 49493	14
28	00123 45678	1	01357 91357	5	02581 47036	9	03715 93715	11	04949 49494	14

Crelle's Journal f. d. M. Bd. XXX. Heft 3.

5: 01234 56789	No.	6: 01234 56789	No.	7: 01234 56789	No.	8: 01234 56789	No.	9: 01234 56789	No.	
05050 50505	15	06284 06284	18	07418 52963	20	08642 08642	24	09876 54321	28	1
05050 50506	15	06284 06285	18	07418 52964	20	08642 08643	24	09876 54322	28	2
05050 50516	15	06284 06295	18	07418 52974	20	08642 08653	24	09876 54332	28	3
05050 50616	15	06284 06395	18	07418 52074	21	08642 08753	24	09876 54432	28	4
05050 51616	15	06284 07395	18	07418 53074	21	08642 09753	24	09876 55432	28	5
05050 61616	15	06284 17395	18	07418 63074	21	08642 19753	24	09876 65432	28	6
05050 61617	15	06284 17396	18	07418 63075	21	08642 19754	24	09876 65433	28	7
05051 61627	15	06285 17306	19	07419 63085	21	08643 19764	24	09877 65443	28	8
05051 61727	15	06285 17406	19	07419 63185	21	08643 19864	24	09877 65543	28	9
05061 62728	15	06295 18407	19	07429 64186	21	08653 10865	25	09887 66544	28	10
05061 62738	15	06295 18417	19	07429 64196	21	08653 10875	25	09887 66554	28	11
05061 72738	15	06295 28417	19	07429 74196	21	08653 20875	25	09887 76554	28	12
05061 72838	15	06295 28517	19	07429 74296	21	08653 20975	25	09887 76654	28	13
05061 72839	15	06295 28518	19	07429 74297	21	08653 20976	25	09887 76655	28	14
05162 73849	15	06396 29528	19	07520 75207	22	08754 21986	25	09988 77665	28	15
05162 73840	16	06396 29529	19	07520 75208	22	08754 21987	25	09988 77666	28	16
05162 73940	16	06396 29629	19	07520 75308	22	08754 21087	26	09988 77766	28	17
05162 83940	16	06396 39629	19	07520 85308	22	08754 31087	26	09988 87766	28	18
05162 83950	16	06396 39639	19	07520 85318	22	08754 31097	26	09988 87776	28	19
05172 84951	16	06306 30630	20	07530 86319	22	08764 32098	26	09998 88777	28	20
05172 84051	17	06306 30730	20	07530 86419	22	08764 32198	26	09998 88877	28	21
05173 84061	17	06307 30740	20	07531 86429	22	08765 32108	27	09999 88887	28	22
05173 84062	17	06307 30741	20	07531 86420	23	08765 32109	27	09999 88888	28	23
05173 94062	17	06307 40741	20	07531 96420	23	08765 42109	27	09999 98888	28	24
05173 95062	17	06307 41741	20	07531 97420	23	08765 43109	27	09999 99888	28	25
05173 95162	17	06307 41841	20	07531 97520	23	08765 43209	27	09999 99988	28	26
05173 95172	17	06307 41851	20	07531 97530	23	08765 43219	27	09999 99998	28	27
05173 95173	17	06307 41852	20	07531 97531	23	08765 43210	28	09999 99999	28	28

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Figures

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4a Bohl [4]
4b Reconstruction by the author

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