Cask Gauging in Germany – without and with Slide Rules (31.07.2019), 11.08.2019

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Kepler's Fassregel (Cask Rule)

When in November 1613 the famous astronomer *Johannes Kepler* married his second wife – his first had died two years earlier – he bought a number of casks of wine. These came from Lower Austria to Linz and were directly sold on the banks of the Danube-river for a reasonable price because of the good vintage that year. After four days the merchant came with his dipping rod to measure the volume in the casks stored in his cellar. Kepler was surprised that this was done without considering the shape of the cask (Fig. 1 is taken from [3]). He doubted that this method could be correct. Fig. 2 demonstrates that two obviously very different casks but with the same length of a dipping rod "d" would give wrong results. Kepler decided to study this problem and to find a more exact and convenient way according to geometric principels [1, 2].

Fig. 1: Mennher, Valentin: Arithmetique seconde (Antwerpen, 1556)

ArithmeumLibrary /Rheinische Friedrich-Wilhelms-Universität Bonn

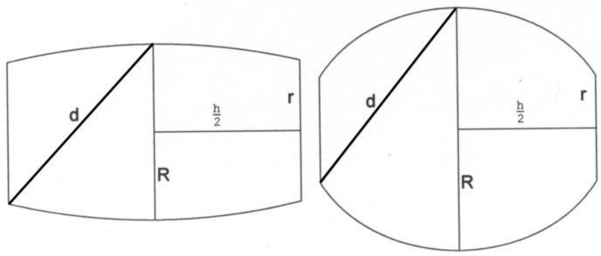


Fig. 2

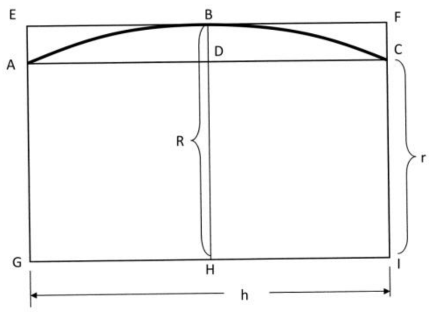
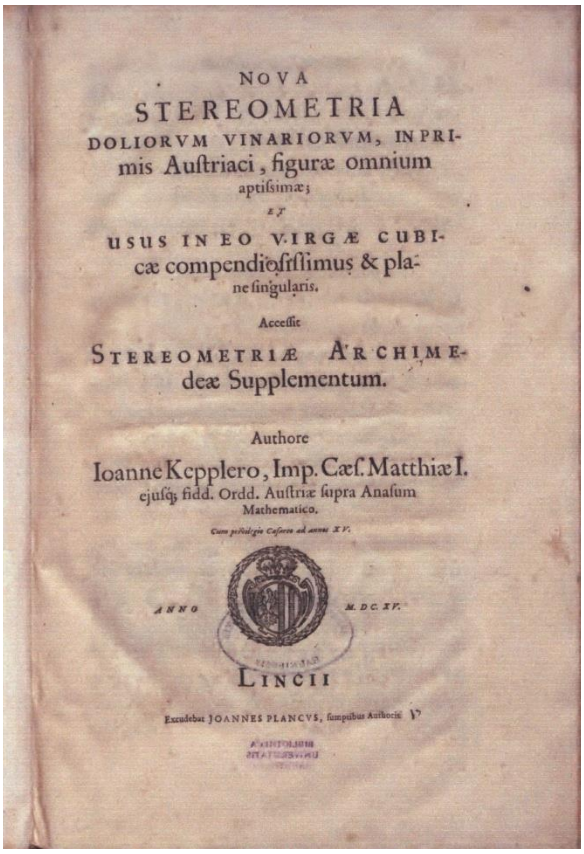


Fig. 3

At this time integral calculus was unknown. So Kepler looked for a simple approximate formula. His idea is shown in Fig. 3. The true value of the area below the cask curvature ABC and the middle line of the cask GHI will be between the two rectangles GEFI and GACI. From the drawing one can see that AEB is smaller than ABD. Kepler concluded that the bung radius BH should be taken twice and the head radius CI only once to come to a more exact value. His simple rule for the volume of a cask is

$$V \approx \tfrac{1}{3}\pi h\,(2R^2 + r^2)$$



NOVA
STEREOMETRIA
DOLIORVM VINARIORVM, INPRI-
mis Auſtriaci, figuræ omnium
aptiſsimæ;
ET
USUS IN EO VIRGÆ CUBI-
cæ compendioſiſſimus & pla-
ne ſingularis.
Acceſſit
STEREOMETRIÆ ARCHIME-
deæ Supplementum.
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Mathematico.
ANNO M. DC. XV.
LINCII
Excudebat JOANNES PLANCVS, ſumptibus Authoris.

Fig. 4

Kepler's rule gives exact values for casks with spherical curvatures and very close values for parabolas. Kepler published his considerations in 1615 in his book NOVA STEREOMETRIA DOLIORVM VINARIORVM (New Volume Calculation of Wine Casks). Fig. 4 shows the title page.

Later authors, like Oughtred in England and Lambert in Germany, used the same formula.

Why do casks bulge?

Everything would be easier if casks were cylindrical. Then calculating the volume and content of partly filed casks would be much quicker and accurate. However, casks bulge because the hoops must press the staves together to make the cask tight. Before this the staves must be bent to the correct shape. Bending is done by using water and heat. According to Johann Friedrich Benzenberg (German astronomer, physical scientist and publisher) [4] the curvature of a cask is defined as *(D-d) : L* or *2B : L* should be at least $^{1}/_{30}$th of the length up to a maximum $^{1}/_{6}$th, in Fig. 5 the ratio 2B: L.

Fig. 5 also shows different shapes of a cask. The broken line represents a cask in the form of a spheroid or the 1st variety (England), resp. Klasse 1 on German slide rules. Klasse 4 (4th variety) is valid for casks consisting of two middle frustums of a cone. Benzenberg's favourite form of a bended stave was the conchoid (in Fig. 5 the dotted line is exaggerated). As wooden staves cannot be bent sharply in the middle, Klasse (variety) 4 will, in practice, not be found.

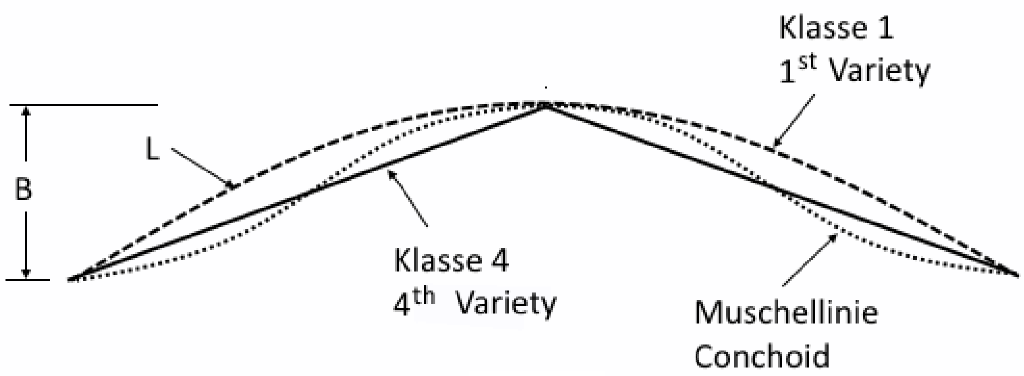


Fig. 5

The bung-/head ratio is determined by the shape of the staves, i.e. the relation between the centre and the ends (B and H in Fig. 6). The dimensions B and H may differ between the staves, but the ratio B: H must be constant for all staves. To achieve this a Cooper uses a gauge/-template. The man in the foreground in Fig. 1 possibly uses such a template. For the curvature of a stave and thus for the variety of the cask the dimensions between B and H (for example "x" or "y" in Fig. 6) are determined. They must also keep the same ratio as B to H.

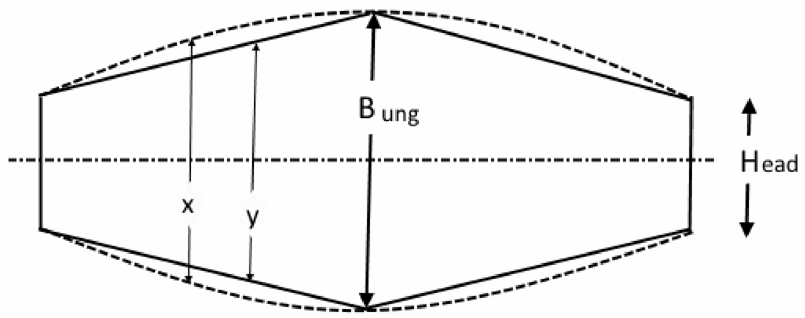


Fig. 6

Varieties

In practice it is not possible for a Cooper to build a cask exactly like a Middle Frustum of a Spheroid (1st variety/ Klasse 1), or of a Parabolic Spindle 2nd variety/ Klasse 2) etc. In order to find a way to calculate the volume of casks in England four varieties were defined and the gauger had to decide which one would the best fit to the relevant cask (Fig. 7a-b):

1st variety = Middle Frustum of a Spheroid

2nd variety = Middle Frustum of a Parabolic Spindle

3rd variety = Middle Frustum of two Parabolic Conoids

4th variety = Middle Frustum of two Cones

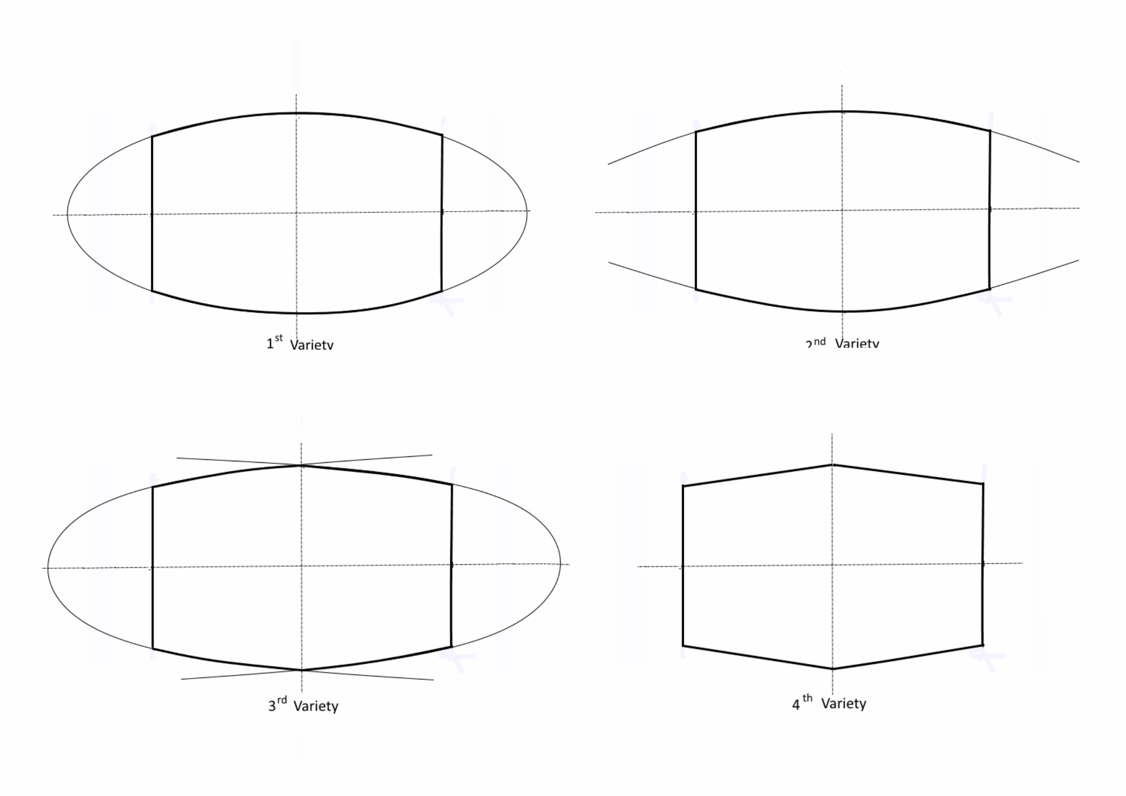
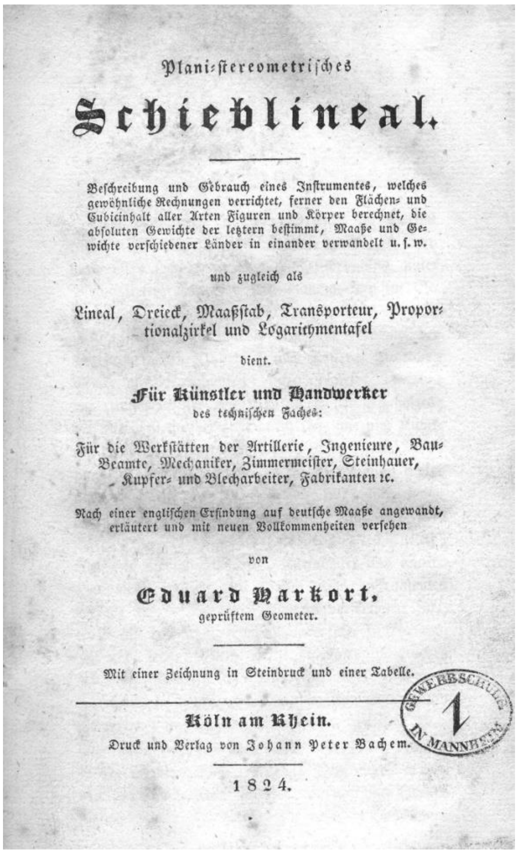


Fig. 7a-b



Plani-stereometrisches

Schieblineal.

Beschreibung und Gebrauch eines Instrumentes, welches gewöhnliche Rechnungen verrichtet, ferner den Flächen- und Cubicinhalt aller Arten Figuren und Körper berechnet, die absoluten Gewichte der letztern bestimmt, Maaße und Gewichte verschiedener Länder in einander verwandelt u. s. w.

und zugleich als

Lineal, Dreieck, Maaßstab, Transporteur, Proportionalzirkel und Logarithmentafel

dient.

Für Künstler und Handwerker des technischen Faches:

Für die Werkstätten der Artillerie, Ingenieure, Bau-Beamte, Mechaniker, Zimmermeister, Steinhauer, Kupfer- und Blecharbeiter, Fabrikanten rc.

Nach einer englischen Erfindung auf deutsche Maaße angewandt, erläutert und mit neuen Vollkommenheiten versehen

von

Eduard Harkort,
geprüftem Geometer.

Mit einer Zeichnung in Steindruck und einer Tabelle.

Köln am Rhein.
Druck und Verlag von Johann Peter Bachem.

1824.

Fig. 8

Casks of the first variety contain more than all others with the same length, head and bung diameter. Casks of the fourth variety contain less than all the others.

As far as it is known today Eduard Harkort [5, pages 130 - 137] was the first author in Germany who proposed a slide rule with rules for the calculation of casks. During a stay in England he had noticed the wide spread use of slide rules in many fields. And he found a description in a small booklet by Andrew Mackay: *Description and Use of the Sliding Rule in Arithmetic and …,* [6]. Mackay's book described a hinged *Coggeshall* rule, a *Ship Carpenter's Slide Rule* and a *4-slide Excise Rule.* Harkort adopted the idea of a hinged rule with a slide in one of the legs. Fig. 8 shows the title page and Fig. 9 a drawing of both faces of his *Schieblineal*. Harkort also adopted the scale arrangement A, B, C, D (D shifted by 4) and some chapters about gauging. On the first side of the second leg he introduced many tables which he thought helpful for users in many applications (Fig. 9). His method for cask calculation will be explained later.

With regard to varieties Harkort also adopted the English custom with 4 varieties, but called them "Klasse".

Klasse 1: Staves are considerably bent

Klasse 4: Staves are straight between bung
and head, i.e. two frustums of cones

Klasse 2 and 3: Intermediate values between
Klasse 1 and 2 determined by
the degree of bending.

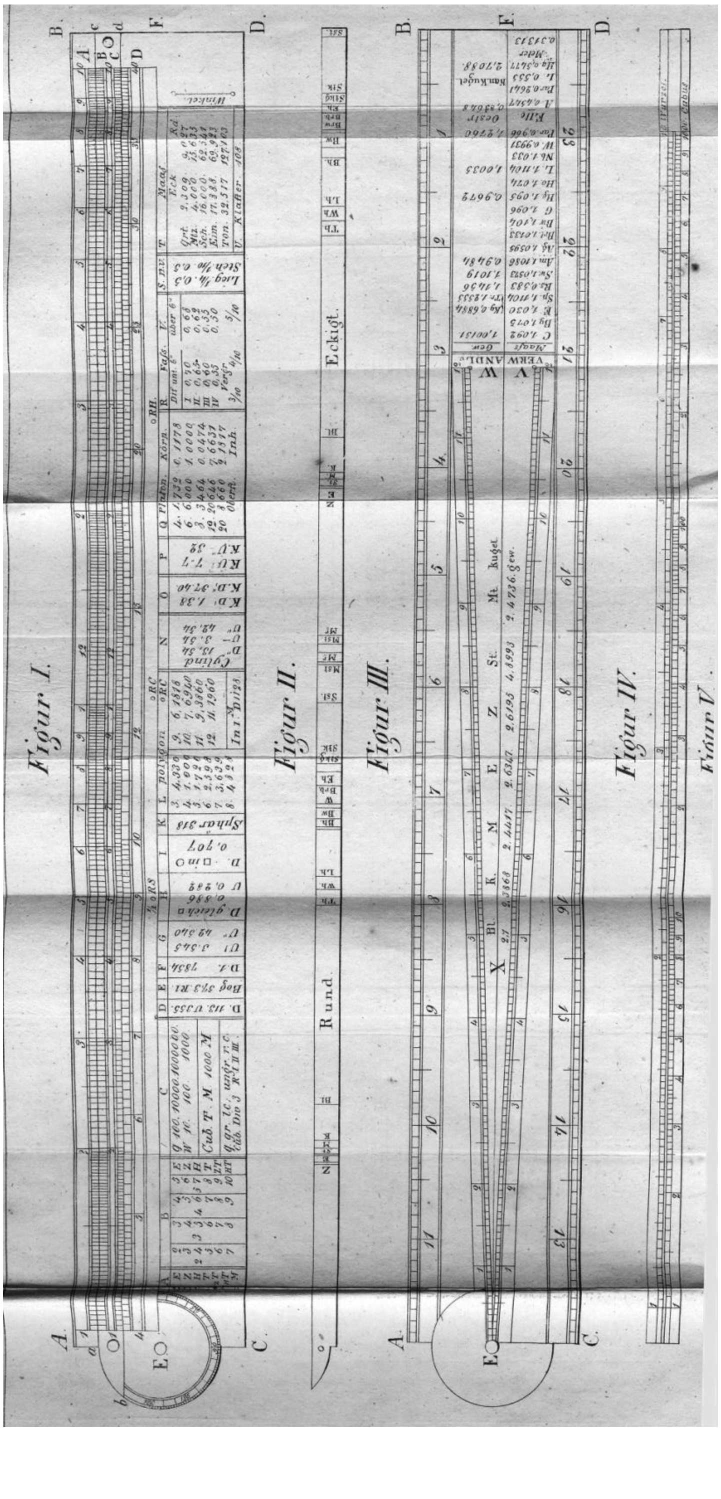


Fig. 9

Three methods to determine the volume and content of casks

1. ***Eichen*** or ***Aichen*** in old German books means ***calibration*** by filling up a cask with water. This should be a very precise method, however, it depends largely on the gauger i. e. on how carefully he fills the liquid measure.
2. ***The stereometrical way*** by taking key dimensions of the cask and calculating by hand or with a slide rule according to various formulas. In Germany this method was rarely used as it was regarded as rather complicated and needed mathematical knowledge.
3. ***Gauging with gauging rods.*** Although rather inaccurate this method was widely used in Germany. Many books and articles have been published about gauging rods.

The subject of this article will be the stereo metrical method.

The stereo metrical method

It was common practice to find the diameter of a cylinder with the same volume as the cask. But the way chosen was quite different between German authors.

Kepler's Fassregel of 1615 was obviously unknown to later authors. Some mentioned Johann Heinrich Lambert (1728 – 1777) who had also used the same formula. It is astounding that even in the early years of the 19^{th} century Benzenberg [4] in 1811 and Bleibtreu [7] in 1833 stated that until the 17^{th} century gaugers used the arithmetic average of head and bung diameter for a cylinder having the same volume as a cask. Later two frustums of cones (i.e. 4^{th} variety or Klasse 4) were taken. Both methods give results with an error of 7 to 10 percent.

Benzenberg and Bleibtreu obviously did not know of the methods proposed by other earlier authors. For example, the 1782 book by Ignaz Pickel, a teacher of mathematics at the academic lyceum in Eichstädt, on the design of *Visierstäbe* (gauging rods) [8] discussed ways of finding the mean diameter of a cask. He stated that the usual way to take the arithmetical average of bung and head diameter gives a 5 percent too low reading. Also using two frustums of cones gives a too small volume of the cask. But the curvature of a cask is more like part of a circle or an ellipsis or a parabola. So he proposed two middle frustums of a parabolic spindle (2^{nd} variety) and his calculation finally gave two similar formulas for the mean diameter: $d_m = d_H + \frac{2}{3} (d_B – d_H) = d_m = \frac{2}{3} d_B + \frac{1}{3} d_H$

$$\text{or } d_m = 0.7\ d_B + 0.3\ d_H$$

Pickel had carefully measured many different casks by filling them with water and found a difference of only 0.35 percent. His main task was to design appropriate *Visierstäbe* (gauging rods). Here he faced the problem of a large number of the different measures for volume, which varied from town to town. However, *Visierstäbe* are not the subject of this article.

Benzenberg studied two casks very carefully: The first was an 8 Ohm (ca. 1100 litre) *Rheingauer Stückfass* with the inner dimensions:

Length = 1490 mm
Bung diameter = 1050 mm
Head diameter = 855 mm

The second, a *Burgundy* cask with the inner dimensions:

Length = 748 mm
Bung diameter = 630 mm
Head diameter = 569 mm

All dimensions are average values of at least two measurements. It is remarkable that as early as 1811 the *Millimetre (Linien)* and *Litre* were used. In the following only the *Rheingauer Stückfass* will be described. It had the shape of a conchoid (Muschellinie), (see Fig. 5). With Lambert's (= Kepler's) formula the diameter of a cylinder with the same volume is:

$$D_{cyl} = \tfrac{2}{3} D + \tfrac{1}{3} d$$
$$= \tfrac{2}{3} * 1050 + \tfrac{1}{3} * 855$$
$$= 700 + 285 = 985 \text{ mm}$$

The calculated volume of the cask: $V = 9{,}85^2 * \pi : 4 * 14{,}9$
$= 1135.4$ litre

By filling the cask with water Benzenberg found the true value was 1145 litre, i.e. the stereo metrical method gave 9.6 litre or 0.85% less. He concluded that the shape was responsible for this difference. A check with a modified formula using the **surface areas** of the bung and the head resulted in a volume of 1145.2 litre and thus exactly the true value.

Later, in 1833 Bleibtreu [7] theoretically examined a cask shaped like a part of a circle. If the staves are not bent too much he found out that the error would only be about 1% if in Lambert's (Kepler's) formula the **surfaces** of the head and bung were used.

In 1824 Eduard Harkort published his *Plani=stereometrischesSchieblineal* [5] which incorporated the instructions of Andrew Mackay's book *Description and Use of the SLIDING RULE …* [6]. For finding the diameter of a cylinder with the same content Harkort had chosen the English way: depending on the difference between bung and head diameter and on the variety a certain number had to be added to the head diameter to find the mean diameter. On the back of one slide of English Excise slide rules one will find lines giving the required number for three varieties, i.e. without the 4^{th} variety which in practice will not be applicable (Fig. 10). Mackay also came up with a formula to calculate the extra amount to be added to the head diameter (Fig. 11).
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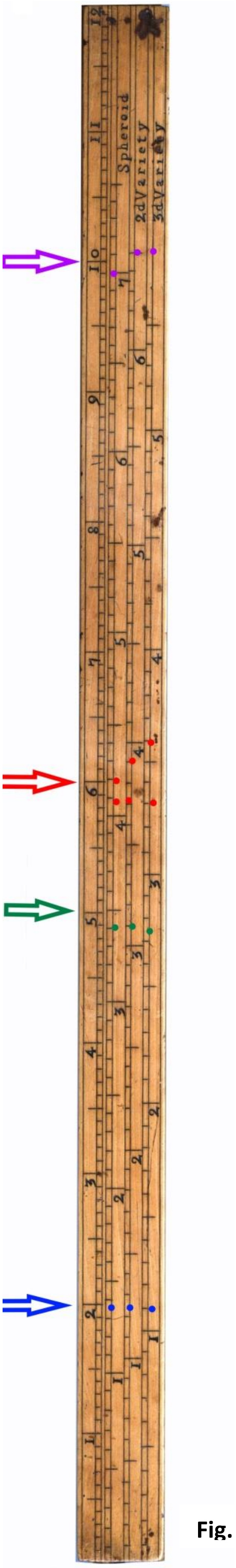


Fig. 10

In order to find the content of a cask, it is previously reduced to a cylinder, by encreasing the head diameter. For this purpose lines are placed on the rule, adapted to the several varieties of casks, and are to be used as follows. Find the difference between the bung and head diameter on the line of inches, and opposite thereto, on the line answering to the variety of the cask, is a number, which added to the head diameter, the sum will be a mean diameter, or that of a cylinder of the same length and capacity as the given cask. Or if the difference between the bung and head diameters, be multiplied by .7 for the first variety; .65 for the second; .6 for the third; and .55 for the fourth variety; and the product added to the head diameter, the sum will be the mean diameter of the cask. In those casks where the difference between the bung and diameters, is less than six inches, it is customary to use the following multipliers, viz: .68 for the first variety; .62 for the second; .55 for the third; and .5 for the fourth variety.

Fig. 11

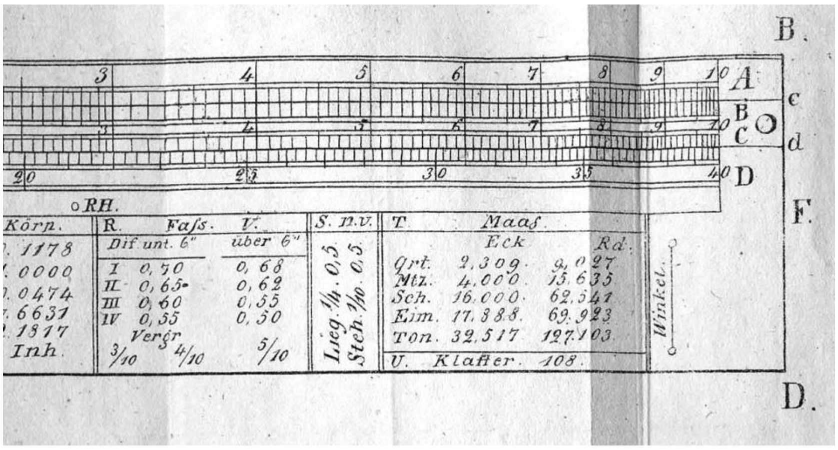


Fig. 12

How to measure the key dimensions of a cask?

To measure the inside bung diameter is easy. It is more complicated to find the correct inside head diameter, which can only be measured from the outside. Depending on the (generally unknown) thickness of the head, the size of the cask and the shape of the cask the inside diameter must be increased by a small amount. We do not find this correction in either Benzenberg or Bleibtreu. However, Harkort, did consider it and again copied the numbers he had found in Mackay's paper, just using the names of similar sized German casks types:

Usual allowance for casks less than 120 Quart (30 Gallons): $^{3}/_{10}$ inch
between 120 and 200 Quart (30 – 50 Gallons): $^{4}/_{10}$ inch
above 200 Quart ((50 Gallons): $^{5}/_{10}$ up to $^{6}/_{10}$ inch

These numbers can be found on Harkort's *Schieblineal* (see Fig. 12).

It is even more difficult to find the correct inside length which can only be measured from the outside. From this the normally unknown thickness of the two heads has to be subtracted. In many cases the heads are thicker in the middle and bevelled at the circumference. Benzenberg suggests ignoring this as the error will be part of the general error. Gaugers usually assumed the thickness of the heads to be the same as the staves.

Oval shaped casks

Sometimes, if the space in a cellar is limited, oval shaped casks were used. It was assumed that the bung and the head surface areas are elliptical. Therefore the mean diameter of circles with the same area is the square root of the product of both axes. Benzenberg and Bleibtreu both gave two ways to calculate the volume of oval casks. Unfortunately, Benzenbergs first method contains significant errors. The second one gives only a formula which Bleibtreu later explained in detail. Fig. 13 shows the dimensions used by Bleibtreu. As the width of a cask at the bung cannot be measured it is assumed that the ratio of the two axes will be same as for the head. Therefore the width will be $^{n}/_{m}$*k.

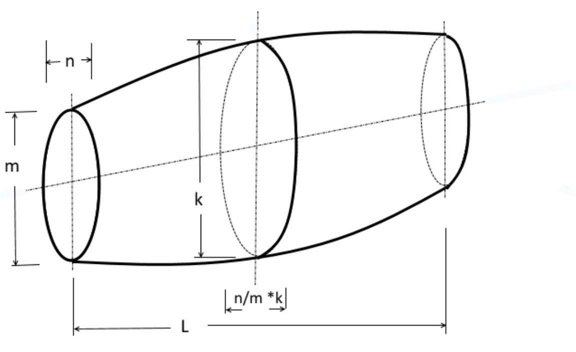


Fig. 13

The diameters of circles with the same area as the ovals are:

$$d_{Head} = \sqrt{m*n} \quad \text{and } D_{Bung} = \sqrt{^{n}/_{m} *k*k}$$

With Kepler's formula we get as mean diameter of the cylinder:

$$d_{Cyl.} = \tfrac{1}{3}\sqrt{m*n} + \tfrac{2}{3}\sqrt{{}^{n}/_{m} * k*k}$$

$$= \tfrac{1}{3}\sqrt{m*n} + \tfrac{2}{3}\, k\sqrt{{}^{n}/_{m}}$$

The volume of the cask is: $V = {}^{\pi}/_{4}\, L\, (\tfrac{1}{3}\sqrt{m*n} + \tfrac{2}{3}\, k\sqrt{{}^{n}/_{m}})^{2}$

$$= {}^{\pi}/_{4}\, L\, ({}^{1}/_{9}\, m*n + {}^{4}/_{9}\, k^{2}\, {}^{n}/_{m} + {}^{4}/_{9}\, k*n)$$

$$= {}^{\pi}/_{4}\, L*\, {}^{n}/_{m}\, ({}^{1}/_{9}\, m^{2} + {}^{4}/_{9}\, k^{2} + {}^{4}/_{9}\, k*m)$$

$$V = {}^{\pi}/_{4}\, L*\, {}^{n}/_{m}\, (\tfrac{1}{3}\, m + \tfrac{2}{3}\, k)^{2}$$

This means that the volume of an oval cask can be calculated with the greater axis of the bung and head and then multiplied with the ratio of the width and the height.

How exact is the stereo metric method?

The above mentioned authors generally stated that, for many reasons an exact volume calculation of a cask is not possible. A difference of 1 to 2% should be accepted by merchants, excise officers and others. If a more exact result is required then they recommend filling a cask with water and measuring the volume. However, this is not straightforward and also still prone to small errors. There are eight reasons for errors:

1. It is difficult to find a correct variety of the cask's shape. This depends very much on the experience of the gauger.
2. Kepler's formula only gives exact results for casks of the 1st variety/ Klasse 1. If the curvature is an arc or like variety 2 or 3, the calculated volume will be exaggerated by 0.5 to 1%.
3. As explained above, the measurement of inside length and head diameters are difficult to take and are often wrong.
4. Nearly all casks are irregular; depending very much on the area where they were built.
5. Most casks are not round on the inside, but a irregular polygon because the Cooper did not shape the staves.
6. All casks will be more or less distorted when filled or after long-term storage. In this case the measurement of the vertical bung diameter will be too small.
7. Tartar on the bottom reduces the measured height.
8. And lastly, already Coopers can be devious. Bleibtreu stated that they make the bottom stave deeper in the middle so that the bung diameter measured will be greater. And they may do the same with the heads so that when measuring the diagonal with a gauging rod, it is longer.

Partly filled casks

Casks may stand horizontal or upright, generally named *lying* or *standing*. Especially lying casks must be absolutely level and horizontal when being measured. Benzenberg (1811) and Bleibtreu (1833) did not deal with standing or with oval casks. Both concentrated on just one cask - the *Rheingauer Stückfass*. The latter referred mainly to Benzenberg's method and the detailed measurement of just that cask.

Also to determine the content of a partly filled cask Benzenberg used the method of a cylinder with the same volume as the cask. This is another fault in his article because the volume of the upper and lower portion of the cask is not being considered (blue areas in Fig. 14).

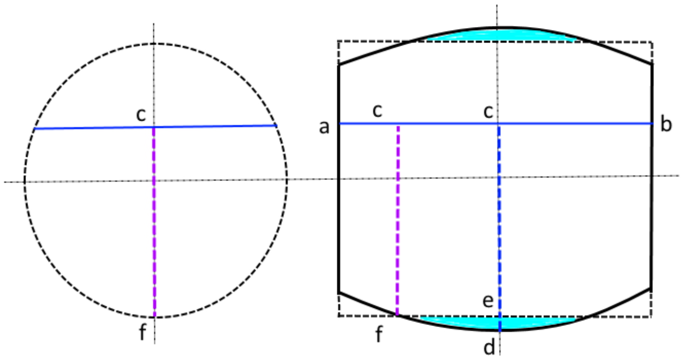


Fig. 14

In the drawing line a – b is the surface of the liquid and c – d is the height measured with the gauging rod. At this time this was called the *Weintiefe* (Wine Depth). c - f was named the *Pfeil* (arrow) and is the *Weintiefe* minus half the difference between the bung diameter and the diameter of the cylinder.

The area below the surface is calculated with the help of the *Segmenttafeln* (Table of Segments) prepared by a *Herr Obereit*. In his book Benzenberg printed all the tables. Fig. 15 shows the page with values used for the following example. The tables are prepared for a circle with a diameter = 1000 and the area = 1. Depending on the length of the *Pfeil* the area of the segment can be found. For the same dimensions as the above described as a *Rheingauer Stückfass* Benzenberg gave an example with a *Weintiefe* = 933 mm.

Länge des Pfeils	Fläche des Segments	Länge des Pfeils	Fläche des Segments	Länge des Pfeils	Fläche des Segments
825	24626	860	0,9149054	895	40943
26	34291	61	57876	96	48733
27	43934	62	66672	97	56490
28	53555	63	75442	98	64213
29	0,8863154	64	84184	99	0,9471904
830	0,8872730	865	0,9192900	900	0,9479560
31	82284	66	0,9201588	1	87182
32	0,8891817	67	10249	2	0,9494770
33	0,8901325	68	18883	3	0,9502324
34	10813	69	0,9227483	4	09843
835	20275	870	0,9236066	905	17328
36	29716	71	44616	6	24777
37	39133	72	53138	7	32190
38	48527	73	61631	8	39568
39	0,8957898	74	70096	9	0,9546910
840	0,8967245	875	78532	910	0,9554216
41	76569	76	86939	11	61485
42	85869	77	0,9295317	12	68718
43	0,8995145	78	0,9303666	13	75914
44	0,9004397	79	0,9311986	14	83071
845	13625	880	0,9320276	915	90192
46	22829	81	28536	16	0,9597275
47	32008	82	36766	17	0,9604319
48	41163	83	44966	18	11325
49	0,9050293	84	53136	19	0,9618292
850	0,9059398	885	61275	920	0,9625220
51	68478	86	69383	21	32109
52	77533	87	77461	22	38958
53	86563	88	85507	23	45767
54	0,9095568	89	0,9393522	24	52535
855	0,9104547	890	0,9401506	925	59263
56	13500	91	09458	26	65949
57	22428	92	17377	27	72595
58	31329	93	25265	28	79198
59	0,9140205	94	33120	29	0,9685759

Fig. 15

Bung diameter: 1050 mm

Mean diameter of the cylinder: 985 mm

½ difference : 65 mm : 2 = 32.5 mm

Weintiefe: 933 mm

Length of Pfeil: 933 – 32.5 = 900.5 mm

Length of Pfeil
for table: 900.5 : 985 = 0.914

Segment area (see Fig. 15): 0.9583

Content of cask: 0.9583 * 1135 = 1087

The true value, found by drawing off water in small quantities from a full cask was 1089 litres, i.e. a difference of only 2 litres or 0.2% (see Fig 16).

Ausmessungen eines nicht vollen Fasses.

Weintiefe unterm Spund	Berechneter Inhalt	Gemessener Inhalt	Unterschied	Weintiefe unterm Spund	Berechneter Inhalt	Gemessener Inhalt	Unterschied
1033		1134		62	1110	1109	1
1025		1133		61	9	8	1
1019		1132		59	8	7	1
1015	1135	1131	+ 4	57	6	6	0
1011	34	30	4	55	4	5	—1
1007	33	29	4	954	3	4	1
1003	32	28	4	53	3	3	0
1000	31	27	4	52	2	2	0
998	29	26	3	51	1	1	0
995	28	25	3	49	0	0	0
93	27	24	3	47	1099	1099	0
90	26	23	3	45	98	98	0
87	25	22	3	44	96	97	—1
85	23	21	2	43	95	96	1
82	22	20	2	41	95	95	0
80	1120	1119	1	940	93	94	1
78	19	18	1	39	92	93	1
75	18	17	1	37	91	92	1
74	17	16	1	35	90	91	1
72	16	15	1	34	88	90	2
970	15	14	1	33	1087	1089	2
69	13	13	0	31	87	88	1
67	12	12	0	30	85	87	2
65	11	11	0	29	84	86	2
64	10	10	0	927	83	85	2

Fig. 16

Benzenberg had drained the full cask in steps: first 50 litres in quantities of 1 litre, then 300 litres in quantities of 5 litres and finally 500 litres in amounts of 10 litres. He stopped taking measurements when the cask still contained 300 litres. Fig. 16 shows only the first of three tables with underlined figures of his above example. After drawn off 1 litre – content now 1134 litres - he measured a *Weintiefe* (Wine Depth) of 1033 mm, 17 mm less than the bung diameter. He gave calculated values only from a *Weintiefe* (Wine Depth) of 1015 mm, i.e. 982.5 mm for the cylinder, very close to the diameter of 985 mm. As explained before his method of using a cylinder for partly filled casks for the volume calculation is wrong. This explains why he gave no calculated values for *Weintiefen* (Wine Depths) above 1015 mm, and that the differences between calculation and measurement become greater if the *Weintiefe* comes close to the diameter of the circle.

Later in his book Benzenberg gave figures for small *Weintiefen*: 17 mm: content 1 litre
25 mm: " 2 litre
31 mm: " 3 litre
35 mm: " 4 litre
39 mm: " 5 litre etc.

Cask calculation with Harkort's Schieblineal

Harkort's *Plani=stereometrisches Schieblineal* based on English hinged rules with the D-scale shifted by "4" did not have special scales or gauge marks to determine the content of casks. The volume calculation for a cask is done on the C and D-scales with the help of the gauge mark 1.13 representing $\sqrt{^4/_\pi}$. Harkort gave the following example for a cask with the variety/ Klasse 1 (see also Fig. 17):

Bung diameter	19"
Head diameter	16"
Difference	3" * 0.7 = 2.1 (see table R on the second leg, Fig. 17)
Mean diameter of cylinder	18.1"
Length	21 "

Fig. 17 shows the solution on the *Schieblineal as* approximately 5400 cubic inches. It has to be remembered that D is a single radius scale shifted by "4" and C is a double radius scale. And one should also note that C normally would be placed on the stock and D on the slide. The correct result for the volume is 5,403.39 cubic inches.

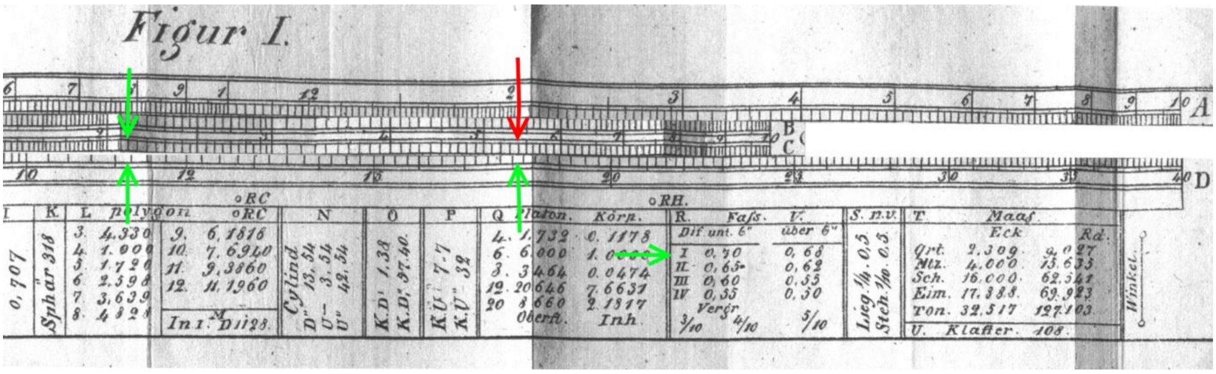


Fig. 17

To find the ullage of lying or standing casks Harkort again used Mackay's alternative solutions "without the rule" (table "S" on Fig. 17). For **lying casks** the rule is:

1. Divide the Weintiefe /Wine Depth (WT) by the bung diameter d_B
2. Add or subtract 0.5 from the quotient
3. Divide the sum by "4"
4. Add $^{WT}/d_B$ to this quotient
5. Multiply the new sum (ullage factor) with the volume of the full cask.

In our language: **Corr. Factor = $^5/_4 * {}^{WT}/d_B$ +(-) 0.125**

For the above example with a *Weintiefe* of 14” the ullage is:

$$\text{Ullage} = \left({}^{5}/_{4} * {}^{14}/_{19} - 0.125\right) * 5403$$
$$= (1.25 * 0.7368 - 0.125) * 5403$$
$$= (0.9211 - 0.125) * 5403$$
$$= 0.7960 * 5403$$

Ullage = 4301 cubic inches

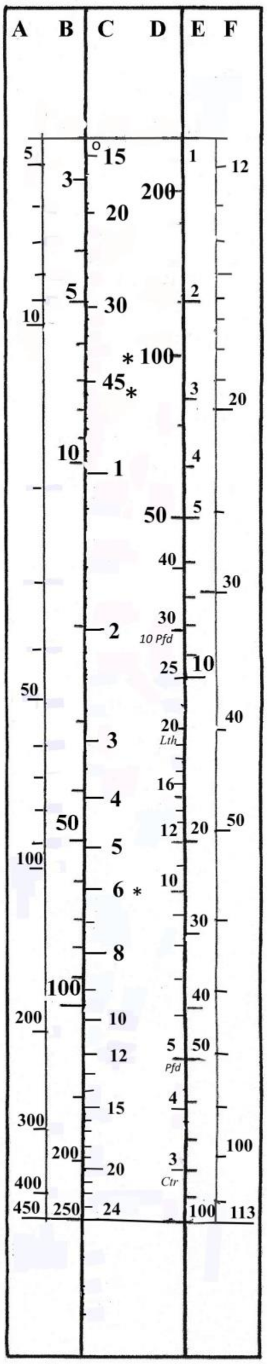


Fig. 18

On the *Schieblineal* the multiplication “Correction Factor * Volume” of the cask can be carried out on the A and B scales. Harkort has checked this procedure with Benzenberg’s measurements of the *Rheingauer Stückfass* and found a very good match.

For the ullage of **standing casks**:

$$\textbf{Corr. Factor} = {}^{11}/_{10} * {}^{WT}/_{L} +(-)0.05$$

Harkort made one *Schieblineal* for his own use and to demonstrate to interested parties. It is very doubtful that anymore were ever fabricated although Harkort did offer them in his book for 2 or 3 *Prussian Thaler*. He was an unsteady and restless man with a lot of other interests [9].

Johann Georg Stökle’s first Polymeter

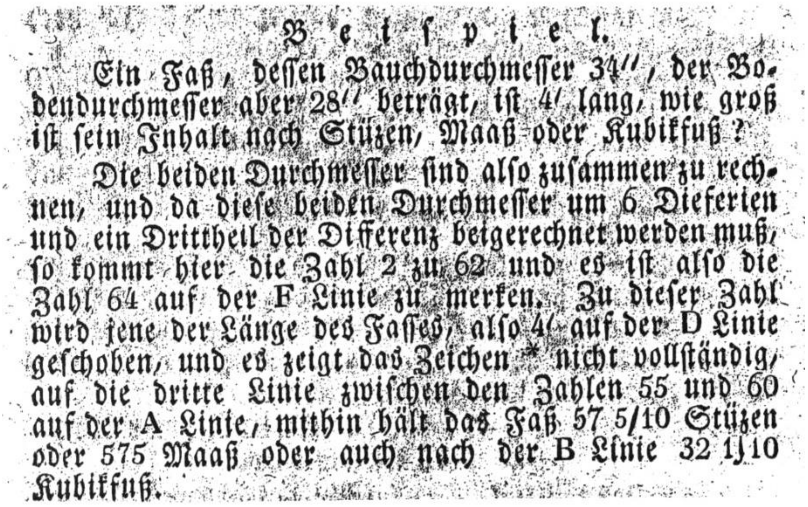
In 1838 J. G. Stökle invented his first *Polymeter,* a complicated arrangement of scales inspired by Michael Eble’s *Dendrometer* [10, 11]. It is a wooden instrument with a glued-on paper strip, 1150 mm long. Divisions and numbers are very accurate but most probably made by hand. All the numbers are written sideways – a custom of old German slide rules – which means that the instrument should be held and used uncomfortably in a vertical position. It is not known how many *Polymeter*s of this type were made. Up to now only one is known to have survived.

A schematic drawing of the *Polymeter* (Fig. 18) shows the sophisticated arrangement of the scales.

In Stökle's limited directions for use there is one example for cask gauging (Fig. 19). The dimensions of the cask are:

Bung diameter	34''
Head diameter	28''
Length	4'

A variety is not mentioned. Required is the content in old German measures (but modified to "Litres"): *Stützen* (15 l), *Maaß* (1.5 l) and Cubic feet.



Beispiel.

Ein Faß, dessen Bauchdurchmesser 34'', der Bodendurchmesser aber 28'' beträgt, ist 4' lang, wie groß ist sein Inhalt nach Stützen, Maaß oder Kubikfuß?

Die beiden Durchmesser sind also zusammen zu rechnen, und da diese beiden Durchmesser um 6 Dieferien und ein Drittheil der Differenz beigerechnet werden muß, so kommt hier die Zahl 2 zu 62 und es ist also die Zahl 64 auf der F Linie zu merken. Zu dieser Zahl wird jene der Länge des Fasses, also 4' auf der D Linie geschoben, und es zeigt das Zeichen * nicht vollständig auf die dritte Linie zwischen den Zahlen 55 und 60 auf der A Linie, mithin hält das Faß 57 5/10 Stützen oder 575 Maaß oder auch nach der B Linie 32 1/10 Kubikfuß.

Fig. 19

The instructions are to add the two diameters and count up one- third of their difference. This gives 34 + 28 + 2 = 64. Place this number on the F-scale opposite to the length in feet on the D-scale (see Fig. 20). At the mark * in the middle of the *Polymeter* the results are found on the A-scale = 57 $^{5}/_{10}$ *Stützen* or 575 *Maaß* and on the B-scale = 32 $^{1}/_{10}$ cubic feet.

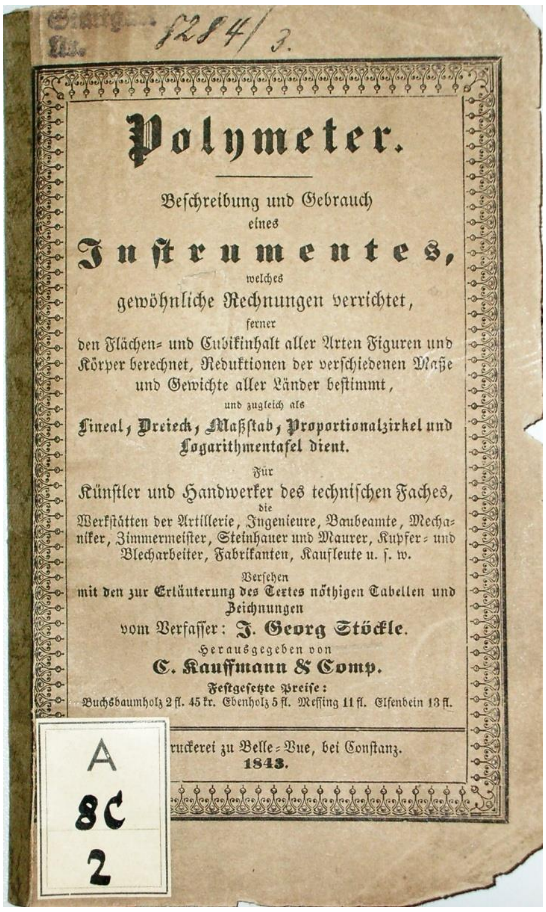
Remarks: Stökle's arrangement of the scales was very sophisticated and with displacements - replacing some multiplications/ conversions. The exception is the F- scale which is a single radius one shifted by $\sqrt{4/\pi}$. All the others are double radius and in addition the D-scale is inverted. His formula for the mean diameter complies with Kepler's resp. Lambert's rule, but is doubled. The foot contains 10 inches with 30 mm each.

Fig. 20

Johann Georg Stöckle's second Polymeter

In 1843 Stöckle, now with a "ck" in his name, invented a completely different *Polymeter* [12]. He now lived in Kreuzlingen on the Swiss side of Lake of Constance. The title page of his instructions for use is shown in Fig. 21.
But this new *Polymeter* was not his own invention; it was a copy of Harkort's *Schieblineal* of 1824. Even the manual was mostly copied and Harkort's name is never mentrioned. Stöckle's only contribution was that he altered Harkort's *Prussian* measures into those of *Baden/ Switzerland* [9].

Quite a few *Polymeters* made by Stöckle can be found in German and Swiss museums and in private collections. They were mostly made by different companies:


Polymeter.

Beschreibung und Gebrauch
eines
Instrumentes,
welches
gewöhnliche Rechnungen verrichtet,
ferner
den Flächen- und Cubikinhalt aller Arten Figuren und Körper berechnet, Reduktionen der verschiedenen Maße und Gewichte aller Länder bestimmt,
und zugleich als
Lineal, Dreieck, Maßstab, Proportionalzirkel und Logarithmentafel dient.
Für
Künstler und Handwerker des technischen Faches,
die
Werkstätten der Artillerie, Ingenieure, Baubeamte, Mechaniker, Zimmermeister, Steinhauer und Maurer, Kupfer- und Blecharbeiter, Fabrikanten, Kaufleute u. s. w.
Versehen
mit den zur Erläuterung des Textes nöthigen Tabellen und Zeichnungen
vom Verfasser: J. Georg Stöckle.
Herausgegeben von
C. Kauffmann & Comp.
Festgesetzte Preise:
Buchsbaumholz 2 fl. 45 kr. Ebenholz 5 fl. Messing 11 fl. Elfenbein 13 fl.

ruckerei zu Belle-Vue, bei Constanz.
1843.

Fig. 21

- POLYMETER • FABRIK • VON • STÖCKLE
- POLY-METER-FABRIK in KREUZLINGEN
- POLYMETER-FABRICATION.VON.J.WACKER . in. EMMISHOFEN
- POLYMETER FABRIK von C: KAUFMANN ET COMP. IN KREUZLINGEN
  Zusatz: OBRIGKEITLICH GEPRÜFT

The earliest known *Polymeter* of the new generation, possibly a prototype, is in the small Museum Rosenegg in Kreuzlingen. It is signed and dated 1844 (Fig. 22a, b, c) and made of brass. Unfortunately, the slide is missing. In the same museum there is also a pocket version made from boxwood (Fig. 23 a, b). The manual also offers *Polymeters* in ivory and in ebony. In 2007 a *Polymeter* in ebony with the slide of German silver was sold at David Stanley in England.

The title page the manual lists the following sales prices:

- Boxwood 2 fl 45 kr (2 Gulden and 45 Kreuzer)
- Ebony 5 fl
- Brass 11 fl
- Ivory 13 fl

Remark: Stöckle's examples in the manual mention 24½ kr for 1 pound of sugar.

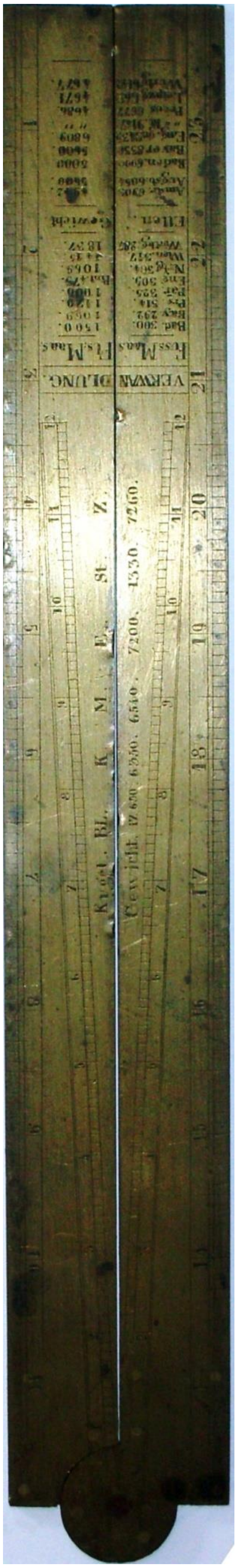


Fig. 22a, b, c

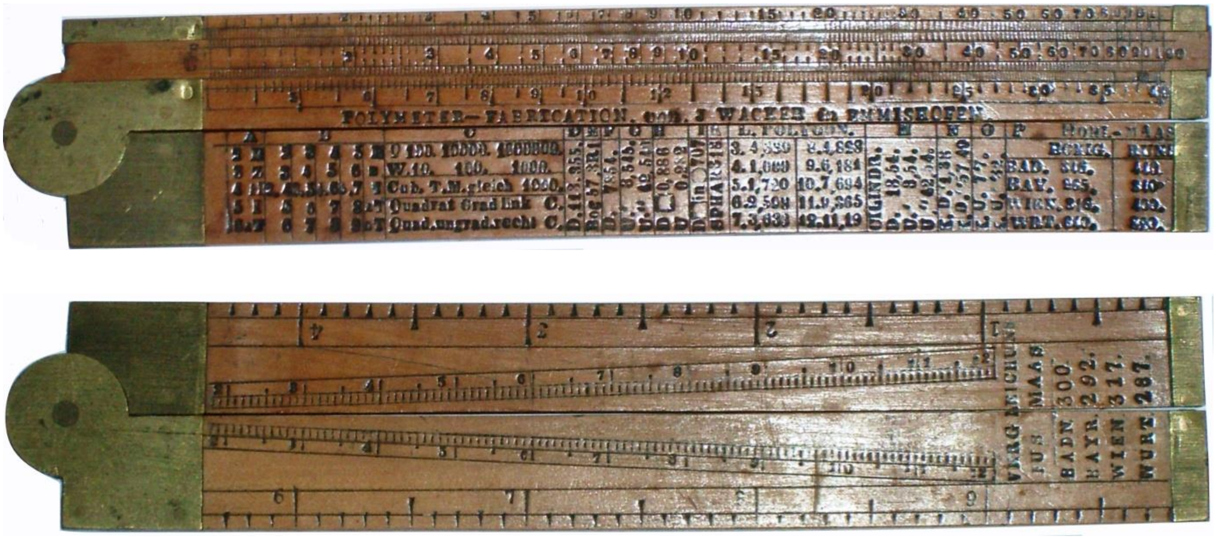


Fig. 23a, b

Stöckle' last manual appeared in 1849 in Elberfeld, today part of Wuppertal. Only a few years later, in 1851, B. Knipp and E. Leisse published a manual for a *POLYMETER or Rechnungs-maßstab* [13]. The authors had obviously based this manual on Stöckle's. There are several *Polymeters* known to be in museums and private collections which fit the 1851 description (Fig. 24 a, b). They are all unsigned and differ from Stöckle's design in some details. These *Polymeters* are designed for the Prussian system of measurements.

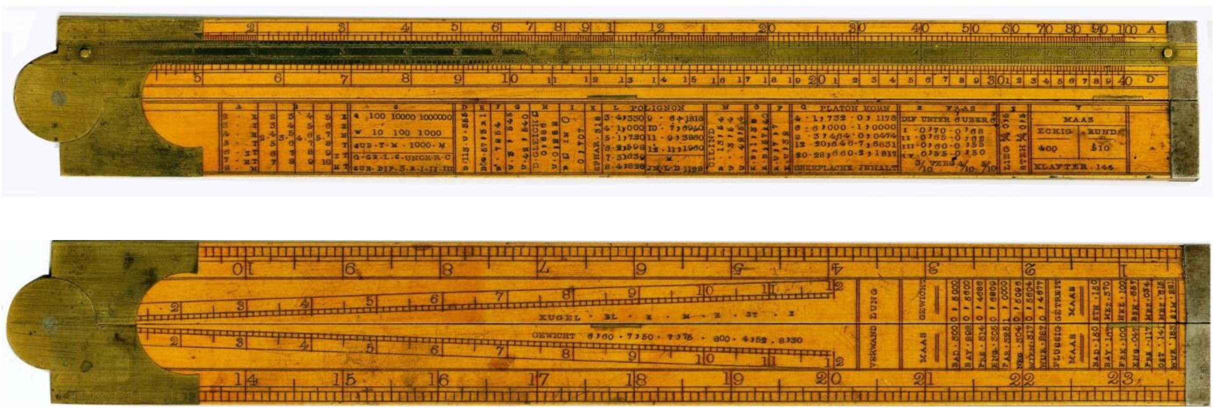


Fig. 24a, b

A comparison with English Excise Officer's slide rules

By 1684 Thomas Everard had already invented the first slide rule especially designed for cask gauging. Over a period of nearly two and a half centuries his slide rule was improved and altered. In total most probably tens of thousands were made in England, mainly for excise officers. Quite a lot can be found in museums but most are in private collections.

In Germany no slide rule was ever made purely for cask gauging. As described above only the *Schieblinea*l by Harkort and the different designs of Stöckle's *Poymeter* bear instructions for gauging a cask but without any special scales. However, the gauging of a cask was equally important in Germany. The usual way was to use *Visierruten* (gauging rods), a rather inaccurate method which Johannes Kepler had criticized back in 1613. Many German authors such as Stephan Weiss have written about *Visierstäbe* or *Visierruten* (gauging rods) [14].

Acknowledgements:

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